Econ 713 Midterm

UW-Madison

Vernal Equinox ♣, March 20, 2023

♣ *There are **100 points in 150 minutes.** (One point a minute is doable). Points are at right.*

♣ **Justify everything with graphs or algebra or a known theorem or class logic.** *Enjoy!*

1. Assume perfect competition for amusement rides, with a rising linear supply $S(p)$. **[15 total]**

   People ride if their enjoyment value v exceeds the expected loss of life value plus the price p. The death chance is $\pi \geq 0$. Someone with income $I > 0$ has life value $L = \beta I$ and ride value $v = \gamma I$, with $\gamma > \beta\pi$. Incomes of potential riders are distributed on $[1-M, 1+M]$, with constant density $R/(2M) > 0$, where $M \in (0,1)$. (In other words, the total mass of potential riders is R.)

   (a) If we learn that rides are more deadly (π rises), does the value of life of the marginal rider rise or fall? What about the average rider? Illustrate this in a simple diagram. **[5 points]**

   (b) What happens to the demand elasticity as M increases? **[5 points]**

   (c) Would supply volatility (a shift in supply curve) produce greater or smaller volatility for ride bought with M bigger? Argue intuitively with your graphical logic instead of algebra. **[5 points]**

   Hint: Note that $1 - M > 0$, so there is a strictly positive price $p \in (0, 1-M)$ for which every consumer buys.

   Solutions: (*a*) By implicit market logic, one rides iff $v - \pi L \geq P$, namely, iff $(\gamma - \beta\pi)I \geq P$. So the highest incomes ride. Let I be the least income that rides,

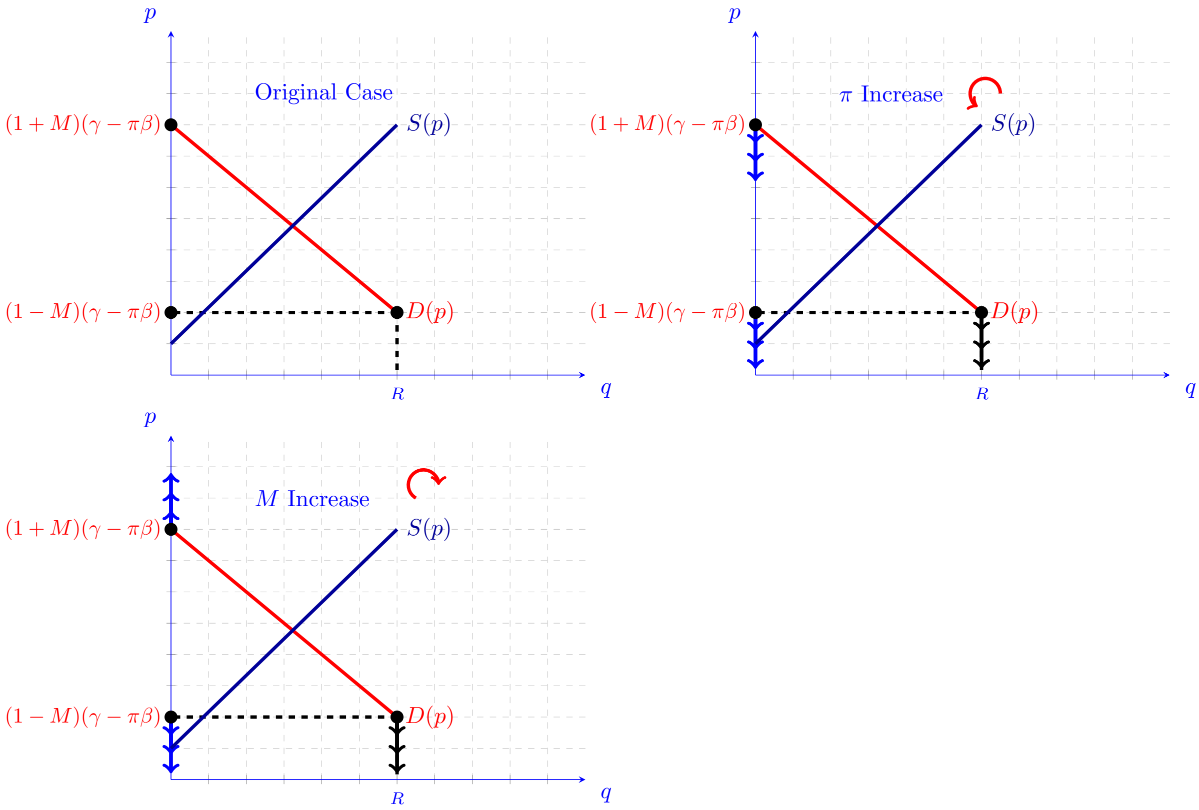
$$D(p) = \int_{I}^{1+M} \frac{R}{2M}\,dx = \int_{P/(\gamma-\beta\pi)}^{1+M} \frac{R}{2M}\,dx = \frac{R}{2M}\left[(1+M) - \frac{P}{(\gamma-\beta\pi)}\right]$$

   However, note that by assumption $(\gamma - \beta/\pi) > 0$ and since $(1-M) > 0$, this only hold for large enough P. In particular, there is a positive price for which everyone rides $\underline{P}$, that is the price that let the lowest income rider indifferent,

$$(1-M)(\gamma - \beta\pi) = \underline{P}$$

   Altogether, the demand is given by

$$D(p) = \begin{cases} \dfrac{R}{2M}\left[(1+M) - \dfrac{p}{(\gamma-\beta\pi)}\right] & \text{if } p > \underline{P} \\ R & \text{if } p \leq \underline{P} \end{cases}$$



(a) As π increases the upper part of the demand falls at rate $(1+M)\beta$, meanwhile the low point of the demand falls at rate $(1-M)\beta$. Thus, the demand not only falls, but also tilts a bit to the left, and so it became more flatter, i.e more elastic. Since both changes makes the demand fall, there are fewer rides in equilibrium, as only the most wealthy people are able to enjoy the raid enough. This lead to an increase in income of the marginal raider and of the average raider.

(b) As M increases, the upper part of the demand increases. Meanwhile the lowest part decreases (both at the same rate $(\gamma-\pi\beta)$). Thus, the demand tilts to the right since the lowest part is fix at R. This makes the demand more inelastic

$$\varepsilon = -\frac{\partial D(p)}{\partial p}\frac{P}{Q} = \frac{P}{(1+M)(\gamma-\beta\pi)-P}, \quad \text{decreasing in } M$$

(c) A steeper linear demand is more inelastic. So we should expect a smaller change in rides.

2. Three fishing companies lie along a river, **[15 total]**

named for their river location: (U)upstream, (M)idstream, and (D)ownstream. Each sells their fish in a competitive market at fixed price p. Production costs of firms M and D depend on the production of firms upstream to them:

$$c_U(q_u) = q_u^2, \quad c_M(q_m) = q_m^2 + q_u q_m, \quad \text{and} \quad c_D(q_d) = q_d^2 + (q_u + q_m)q_d$$

(a) Find the Nash equilibrium production levels if they choose simultaneously. **[3 points]**

(b) Intuitively explain why the equilibrium is socially inefficient. How would a social planner adjust firm outputs? **[4 points]**

(c) Could a single Pigouvian fishing tax solve the problem? if so, find the tax. If not, why? **[5 points]**
Hint: Just reason intuitively, and do not solve any maximizations.

(d) Assume Coasian bargaining, where firms have the right to produce any output. Who transfers to whom? **[3 points]**
Hint: You may reason intuitively having solved (a) and (b), without doing more math.

Solutions:

(a) In a Nash equilibrium, q_U, q_M, and q_D solve the three maximizations:

$$\max_{q_U} \; pq_U - q_U^2 \quad \text{and} \quad \max_{q_M} \; pq_M - q_M^2 - q_U q_M \quad \text{and} \quad \max_{q_D} \; pq_D - q_D^2 - q_U q_D - q_M q_D$$

The FOCs are

$$p = 2q_U \quad \text{and} \quad p = 2q_M + q_U \quad \text{and} \quad p = 2q_D + q_U + q_M$$

Thus, $q_U = p/2$, $q_M = p/4$, and $q_U = p/8$.

(b) Nash equilibrium is intuitively socially inefficient, since upstream firms ignore their harm on downstream, and so overproduce, which reduces downstream output. Socially efficiency requires

$$\max_{q_U, q_M, q_D} \; p(q_U + q_M + q_D) - q_U^2 - q_M^2 - q_D^2 - q_U q_M - q_U q_D - q_M q_D$$

The FOCs are

$$p = 2q_U + q_M + q_D \quad \text{and} \quad p = 2q_M + q_U + q_D \quad \text{and} \quad p = 2q_D + q_U + q_M$$

Symmetric equations admit a symmetric solution: $q_U = q_M = q_D = p/4$. So firm U should produce more than in equilibrium and D less, while M is unchanged.

(c) A Pigouvian tax would not fix the problem, since it raises all firms' costs equally, but we must reduce the upstream firm's output and increase the downstream firm's output.

(d) With well defined property rights, and Coasian bargaining, the efficient outcome emerges.

A first stab might say the downstream D pays upstream U to produce less, while midstream M does not adjust and does not want to adjust since M is best replying to U in the planner solution. But this misses the adjusted incentives of M when U produces less — M wants to produce more. So his outside option (when deviating from the planner solution) is really to demand a transfer too.

So what can we say about transfers? They will sum to 0 and be bounded by what each firm can get by deviating from the planner solution. We invite you to ponder what that set is, and if you do, we can modify the solution to include this math. :)

3. People are fed up with "price gouging" during hurricanes. Florida passes a law mandating that bottled water not increase in prices during hurricanes. When Hurricane Rodrigo hits, the demand for bottled water leaps up. Without the law, the price would have jumped up by \$5 a bottle. People engage in costly queuing in order to purchase ten bottles of water (the maximum allowed) at its fixed price. Assume people precisely estimate average queue time, and no one starts lining up and then quits. What can you say about the total deadweight loss of the law if one million bottles are bought in the hurricane, and *crucially, people are heterogeneous in their hourly cost of queuing.* **[5 total]**

   Solution: The marginal person who queues pays ten times \$5 or \$50 of queue time. *But the inframarginal person pays less.* Deadweight loss is thus **strictly less than** $100,000 \times \$50 = \$5M$. So the key points here are that deadweight loss is the whole rectangle and not just a triangle, but that only the marginal queuer pays that cost, and inframarginal ones pay less.

4. Two airlines $\mathcal{A} = \{a_1, a_2\}$ compete. Each airline a_i has a cost c_i and a capacity k_i, where **[<u>20 total</u>]**

$$c_1 = 85, \quad k_1 = 2, \quad c_2 = 150, \quad k_2 = 3$$

The capacity is the maximum number of passenger that the airline can take in a single flight. The flight the costs are independent of the number of passengers.

If an airlines does not offer a flight, its payoff is zero. There are three potential travelers, $\mathcal{B} = \{b_1, b_2, b_3\}$, respectively willing to pay for a flight:

$$w_1 = 55, \quad w_2 = 60, \quad w_3 = 70$$

Let $V(\mathcal{S})$ be the maximal value of coalition $\mathcal{S} \subseteq \mathcal{I} = \mathcal{A} \cup \mathcal{B}$. Of course, airlines and passengers by themselves generate no value: $V(\mathcal{S}) = 0$ for all $\mathcal{S}$ such that $\mathcal{S} \cap \mathcal{A} = \emptyset$ or $\mathcal{S} \cap \mathcal{B} = \emptyset$.

(a) What coalitions have strictly positive maximal values. What is that value in each case? **[4 points]**

(b) What is the core of this game — namely, all feasible payoffs (profits) x_1 and x_2 for airlines, and travelers' buyer surpluses y_1, y_2, and y_3, blocked by no coalition? **[10 points]**
Hint: One quick trick is to subtract each of four key core constraints (three for airline a_1 and one for airline a_2) from the grand coalition feasibility constraint. Then add up the resulting inequalities, and compare this to the grand coalition feasibility constraint.

(c) Is the value $V(\mathcal{S})$ supermodular? (Equivalently, is this a convex game?) Why? Quote a theorem that allows you a different route to your answer of (b). **[6 points]**

Solutions:

(a) First, the only positive value coalition with airline a_2 is the one with all passengers, and the only positive value coalitions with airline a_1 has two passengers. Altogether, values are:

$$\begin{array}{lll} V(\{a_1, \mathcal{B}\}) = 45 & V(\{a_2, \mathcal{B}\}) = 35 & V(\{\mathcal{A}, \mathcal{B}\}) = 45 \\ V(\{a_1, b_1, b_2\}) = 30 & V(\{a_1, b_1, b_3\}) = 40 & V(\{a_1, b_2, b_3\}) = 45 \\ V(\{\mathcal{A}, b_1, b_3\}) = 40 & V(\{\mathcal{A}, b_2, b_3\}) = 45 & V(\{\mathcal{A}, b_1, b_2\}) = 30 \end{array}$$

(b) But since the class slides only asserted that supermodularity was sufficient in this theorem, we directly prove the empty core using the hint. The core constraints include:

$$x_1 + y_1 + y_2 \geq 30 \ \& \ x_1 + y_1 + y_3 \geq 40 \ \& \ x_1 + y_2 + y_3 \geq 45 \ \& \ x_2 + (y_1 + y_2 + y_3) \geq 35$$

Respectively subtracting each from the grand coalition core constraint $x_1 + x_2 + y_1 + y_2 + y_3 = 45$:

$$x_2 + y_3 \leq 15 \quad \& \quad x_2 + y_2 \leq 5 \quad \& \quad x_2 + y_1 \leq 0 \quad \& \quad x_1 \leq 10$$

Summing, we have $x_1 + 3x_2 + y_1 + y_2 + y_3 \leq 15 + 5 + 10 = 30$. This is not feasible.

(c) We show V is not supermodular. Take $\mathcal{S} = \{a_1, b_1, b_3\}$ and $\mathcal{T} = \{a_1, b_1, b_2\}$,

$$V(\{\mathcal{S} \cup \mathcal{T}\}) + V(\{\mathcal{S} \cap \mathcal{T}\}) = 40 + 0$$

but

$$V(\{\mathcal{S}\}) + V(\{\mathcal{T}\}) = 40 + 30$$

so

$$V(\{\mathcal{S} \cup \mathcal{T}\}) + V(\{\mathcal{S} \cap \mathcal{T}\}) < V(\{\mathcal{S}\}) + V(\{\mathcal{T}\})$$

As we expected, since the core is empty by our result in (b), the supermodularity condition of the Bondareva-Shapley theorem fails.

5. In a Robinson Crusoe economy, a single consumer is endowed with 10 units of labor (ℓ) and 10 units of capital (k). At her disposition are two linear production technologies for final goods x and y: **[25 total]**

$$x = f(\ell, k) = 2\ell + 4k, \qquad y = g(\ell, k) = \ell + 8k$$

Her utility over the goods is $u(x, y) = \log(x) + \beta \log(y)$.

(a) Derive and carefully plot the Production Possibility Frontier (PPF). **[3 points]**

(b) Characterize the equilibrium of this economy for all $\beta \in [0, \infty)$ **[10 points]**

Hint: Draw the indifference curve at the "kink" in the PPF. What is its slope? What does its slope tell you about the social optimum?

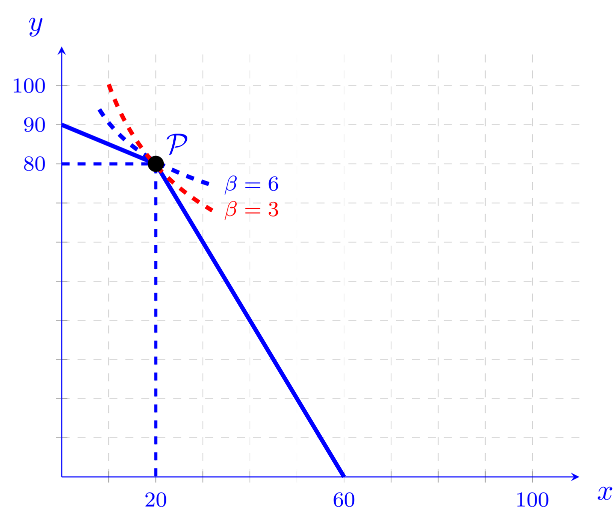
(c) Let x be numeraire. Show that as β rises from zero, the price of y is first constant in β, then rising, then constant again. What is it constant at? **[6 points]**

Hint: Take inspiration from your PPF-indifference curve diagram.

(d) If $\beta = 1/2$, what is the shadow price of labor? **[6 points]**

Solutions:

(a) To obtain the PPF, first produce $x = 60$ and $y = 0$. Then re-allocate the less productive inputs first, in this case capital ($dk < 0$). The PPF is linear in this re-allocation, with slope $dy/dx = -2$, since $dy = -8dk$ and $dx = 4dk$. So $120 - 2x$ on the lower segment. Once all capital has been re-allocated to y, the transfer of labor begins. The PPF is again linear, now with slope $dy/dx = -1/2$. Now, $y = 90 - x/2$ on the upper segment.



(b) The marginal rate of substitution for the consumer is $MRS = \dfrac{y}{\beta x}$. We have drawn red and blue indifference curves tangent to the two PPF line segments. In the Robinson Crusoe economy, the PPF is like a consumer's budget constraint. Production may take place: (i) at the lower PPF segment; (ii) at the kink; (iii) at the upper PPF segment. On the lower PPF segment, since the rate of technical substitution is 2, optimality demands for all $x \in [20, 60]$:

$$\frac{y}{\beta x} = \frac{120 - 2x}{\beta x} = 2 \implies x = \frac{60}{1 + \beta} \Rightarrow y = \frac{120\beta}{1 + \beta}$$

The kink is $x = 20$ and $y = 80$, and so occurs at $\beta = 2$. So for $\beta \in [0, 2]$ the indifference curve is steeper than the PPF at the kink, and so equilibrium is on the lower PPF segment.

Next, on the upper PPF segment, for all $x \in [0, 20]$:

$$\frac{y}{\beta x} = \frac{90 - x/2}{\beta x} = \frac{1}{2} \implies x = \frac{180}{1+\beta} \Rightarrow y = \frac{90\beta}{1+\beta}$$

The kink is $x = 20$, and so $\beta = 8$. So for $\beta \in [8, \infty)$, equilibrium is on the upper PPF segment.

Lastly, for any $\beta \in (2, 8)$ the equilibrium is at the kink $x = 20$ and $y = 80$.

(c) By the consumer optimization, for any $\beta \in [0, \infty)$, we have $p_y = 1/MRS = \beta x/y$. For $\beta \in [0, 2]$,

$$p_y = \frac{\beta x^e}{y^e} = \frac{60\beta/(1+\beta)}{120\beta/(1+\beta)} = 1/2$$

For $\beta \in (2, 8)$,

$$p_y = \frac{\beta x^e}{y^e} = \frac{20\beta}{80} = \frac{\beta}{4}$$

Lastly, for $\beta \in [8, \infty)$,

$$p_y = \frac{\beta x^e}{y^e} = \frac{180\beta/(1+\beta)}{90\beta/(1+\beta)} = 2$$

(d) If $\beta = 1/2$ then $\beta \in [0, 2]$. Here, production is on the lower PPF segment. So the extra labor $d\ell$ optimally produces $dx : dy = 1 : 2$. On the other hand, the production functions give $dx = 2d\ell_x$ and $dy = d\ell_y$. So

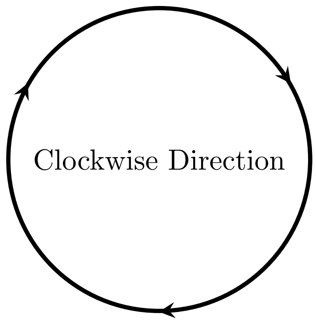
$$2 = \frac{dy}{dx} = \frac{d\ell_y}{2d\ell}$$

Thus, $d\ell_x = d\ell/5$ and $d\ell_y = 4d\ell/5$. This produces $(2/5)d\ell$ units of x and $(4/5)d\ell$ units of y, respectively. This has value $p_x(2/5)d\ell + p_y(4/5)d\ell = 1 \cdot (2/5)d\ell + (1/2)(4/5)d\ell = (4/5)d\ell$, since $p_x = 1$, and using $p_y = p_x/2$. The shadow price of labor is $4/5$.

6. Consider a circular city of perimeter one, with consumer transportation cost $t > 0$ per unit distance, and firm entry cost $f > 0$, and constant marginal cost $c > 0$. We assume consumers are in the continuum, uniformly distributed along the city. They value the good enough so that they always wish to buy it. **[<u>20 total</u>]**

In the symmetric monopolistically competitive long run equilibrium (with free entry), we showed in section that $n^* = \sqrt{t/f}$ firms locate equally spaced around the circle, and charge price $p^* = c + \sqrt{tf}$.

After the symmetric equilibrium has been in place for a while, the city adds traffic lights that make clockwise travel easier than counterclockwise. Specifically, consumers travel to a store in a clockwise direction at unit cost $t/2$, and in a counterclockwise direction at unit cost $2t$.



For these questions, assume the short run, where the firms number n^* and locations are fixed.

(a) Do the firms <u>make more profits</u> after this traffic light addition? **[8 points]**
Hint: Consider scenarios with high and low fixed cost f.

(b) Does the total consumer travel cost fall in the city after the traffic lights are added? **[12 points]**
Hint: Consider high and low unit transportation cost t.

Solutions:

(a) In the short term, there is no entry, or re-locations, so that firms only adjust the price. Exit is allowed in the short run, but only happens if firms cannot pay their marginal costs, which does not happen here, since price exceeds marginal cost.

Since the distance between two stores is $1/n^*$, the traveling distance of the consumers is $x \in [0, 1/n^*]$. But demand from the left and right sides are different. Let's consider first the right side of the demand of firm i (who must travel counterclockwise), when a hypothetical firm j at the right of i charges price p

$$p_i + 2xt = p + \frac{t}{2}\left(\frac{1}{\sqrt{t/f}} - x\right) \quad \implies x_r(p_i, p) = \frac{2p - 2p_i}{5t} + \frac{1}{5\sqrt{t/f}}$$

Next, we do the same to the left side of the demand,

$$p_i + \frac{xt}{2} = p + 2t\left(\frac{1}{\sqrt{t/f}} - x\right) \quad \implies x_\ell(p_i, p) = \frac{2p - 2p_i}{5t} + \frac{4}{5\sqrt{t/f}}$$

Altogether, firm i faces total demand

$$D_i(p_i, p) = x_r(p_i, p) + x_\ell(p_i, p) = \frac{4(p - p_i)}{5t} + \frac{1}{\sqrt{t/f}}$$

Solving firm i's profit maximization,

$$\max_{p_i} \; D_i(p_i, p)(p_i - c) - f$$

We evaluate the FOC at $p_i = p$ to obtain the symmetric equilibrium:

$$\frac{\partial \pi}{\partial p_i} = \frac{1}{\sqrt{t/f}} + \frac{4(p + c - 2p_i)}{5t}\Bigg|_{p_i = p} = 0 \quad \Rightarrow \quad p^* = c + \frac{5}{4}\sqrt{tf}$$

The firm's profit (even paying its fixed cost, usually omitted from the short term profit) is thus:

$$\pi = D_i(p, p)(p - c) - f = \frac{5}{4}f - f = f(1/4) > 0$$

Thus, firms always benefit from this change and the profits are higher as the fix cost increases.
(b) We analyze a single arc of the city, for consumers between two firms:

$$\text{Lights transportation cost} = \int_0^{x_r(p,p)} 2xt \, dx + \int_0^{x_\ell(p,p)} \frac{xt}{2} \, dx = t\left(x_r^2(p,p) + \frac{x_\ell^2(p,p)}{4}\right) = \frac{f}{5}$$

We compare it with the transportation cost without traffic lights,

$$\text{No lights transportation cost} = 2t \int_0^{1/2n^*} x \, dx = t\left(\frac{1}{2n^*}\right)^2 = \frac{f}{4}$$

Thus, total transportation cost falls, independently of the transportation cost.