Econ 713 Final

(Monday, March 21, 2022)

Cite any theorems you apply. Rigorously justify everything.

Assume all values and costs are common knowledge.

There are 120 total points in this exam. Think of 100 as a perfect score. Enjoy!

1) **Beware the ideas of March.** At the start of COVID in early 2020, oil demand fell 10% due to stay at home, and prices fell 40%. After the Russian invasion, oil supply fell 5% and oil prices rose 60%. [10]

Assume that supply did not change in the first case, and demand did not change in the second. Is supply or demand more absolutely elastic? If everyone could use a Tesla *and* a gas car, which price change would have been attenuated and how?

Solution: By the elasticity formula. $\epsilon = -5/60 = -1/12, \eta = 10/40 = 1/4$. *Supply is more absolutely elastic. With a spare Tesla, people can shift into driving it, and demand is more elastic. The second price rise would have been attenuated.*

2) **Some Core Ideas.**

(a) Three business owners approach the owner of a machine. The owner Karen has a single machine to sell, and has an opportunity cost of 20 of selling the machine. One buyer Iris is somewhat inefficient, but has good ideas and so can earn profit 24 by buying the machine. Another buyer Joe is somewhat more efficient – he can earn profit 30 by purchasing the machine. Iris and Joe have a zero outside option. [5]

What is the core of this trading situation (specify allocation and payoffs)?

Solution: Denote by x the seller's payoff and y_I, y_J the buyers' payoffs. 1-person coalitions form the following constraints: $x \geq 20$, $y_I \geq 0$, $y_J \geq 0$. Coalitions with the two buyers together are no better off: $y_I + y_J \geq 0$. Coalitions with a buyer and seller yields $x + y_I \geq 24, x + y_J \geq 30$. Finally, $x + y_I + y_J \leq 30$. Altogether, we see that Joe buys the machine, and so $y_I = 0$, and $x + y_J = 30$, with $x \geq 24$. To summarize, the core payoffs are $\{(x, y_I, y_J); y_I = 0, x \in [24, 30], y_J = 30 - x\}$.

(b) There are three identical chopstick, with owners C_1, C_2, and C_3. Johnny B and Suzy Q like chop sticks. One chop stick is worth 1 to Johnny B and 2 to Suzy Q. Two are worth 10 to Johnny B and 11 to Suzy Q. Find the core. [5]

Solution: We claim that the core is empty. Suppose not. Let respective core payoffs of C_1, C_2, C_3 and Johnny B and Suzy Q be $(x_1, x_2, x_3, y_B, y_Q)$. Since a third chopstick has no value, grand coalition feasibility is (★) $x_1 + x_2 + x_3 + y_B + y_Q \leq 12$. Next, since there are no blocking coalitions, we have

$$[x_1 + x_2 + x_3 - x_i] + y_Q \geq 11 \qquad x_i + y_B \geq 1 \qquad \textit{for } i = 1, 2, 3$$

Summing these yields $x_1 + x_2 + x_3 + y_B + y_Q \geq 12$. Given (★), all inequalities are tight. Hence, $x_1 = x_2 = x_3$ and $y_Q = y_B + 1$. Now, $2x_1 + y_B + 1 = 11, x_1 + y_B = 1$ implies $x_1 = 9$ and $y_B = -8$. This contradicts $y_B, y_Q \geq 0$, valid since Johnny B and Suzy Q need not trade.

3) **Beach Day!** A unit mass of consumers willing to buy a single cup of lemonade populates a circular beach around a lake (uniform density), with circumference 1. Utility is quasi-linear, with utility increment $u(p, x) = a - p - t|x|$ from buying lemonade at price p from a pushcart distance x away, for transportation cost $t > 0$. Assume $a > 0$ is large enough that consumers buy the lemonade. Pushcarts have costs cq for a quantity q of lemonade (where $c > 0$) and pay a beach fee $\phi > 0$. [15]

In the simplest monopolistic equilibrium, what happens to the number of pushcarts and the lemonade price as the beach fee ϕ rises? [Just ignore integer problems.]

Solution: We focus on the symmetric equilibrium where N pushcarts spread out equally and charge the same price p. If a pushcart (normalize his location as 0) charges a possibly different price $\hat{p}$, then consumers at locations $x_0(\hat{p}) < 0 < x_1(\hat{p})$ are resp. indifferent between buying at the pushcarts at $\mp 1/N$ if and only if:

$$a - \hat{p} + tx_0(\hat{p}) = a - p - t(x_0(\hat{p}) + 1/N) \text{ and } a - \hat{p} - tx_1(\hat{p}) = a - p - t(1/N - x_1(\hat{p}))$$

This implies a demand from consumers in $[x_0(\hat{p}), x_1(\hat{p})]$, and so a demand curve

$$D(\hat{p}|p, N) = \frac{p - \hat{p} + t/N}{2t} - \frac{\hat{p} - p - t/N}{2t} = \frac{p - \hat{p} + t/N}{t}$$

This pushcart's post entry profit maximization problem is therefore:

$$\pi(N) = \max_{\hat{p}} (\hat{p} - c) D(\hat{p}|p, N) = \max_{\hat{p}} (\hat{p} - c) \left(\frac{p - \hat{p} + t/N}{t} \right)$$

Since the FOC is solved at $\hat{p} = p$, we have $(\hat{p} - c)[p - \hat{p} + t/N]$, we have

$$[p - \hat{p} + t/N] - [\hat{p} - c]\Big|_{\hat{p}=p} = 0 \quad \Rightarrow \quad p = c + t/N \quad \Rightarrow \quad \pi(N) = t/N^2$$

So pushcarts enter until $\pi(N^*) = \phi$, and so $N^* = \sqrt{t/\phi}$. The number of pushcarts N falls, and the market price p^* rises, as the square root of the beach fee ϕ.

4) **Positive externalities.** Three neighbors, Drake, Josh, and Megan, have to shovel their own snow after a major snowfall. They get a personal benefit from shoveling x cubic feet of snow in front of their house, $B(x) = 10x$ and a secondary benefit from the amount of snow y their neighbors have shoveled, $S(y) = 2y$. Assume that shoveling snow is taxing, and comes with cost $C(x) = x^2$. [15]

(a) Suppose no negotiation is possible. How much snow will each person shovel?
Solutions: Every x selfishly maximizes $u(x) = B(x) + S(y) - C(x) \Rightarrow x = 5$.

(b) Suppose Coasian negotiation is possible. How much more snow will each person shovel? Describe all possible outcomes from Coasian bargaining.
Solution: The social objective function is:

$$\begin{aligned} W &= \textstyle\sum_i [B(x_i) + S(y_i) - C(x_i)] \\ &= 10(x_D) + 2(x_J + x_M) - x_D^2 + 10(x_J) + 2(x_D + x_M) - x_J^2 + 10(x_M) + 2(x_D + x_J) - x_M^2 \\ &= \textstyle\sum_i [14x_i - x_i^2] \end{aligned}$$

The FOC yields $14 - 2x_i = 0 \Rightarrow x_i = 7$, *and so more shoveling by 2.*

Let T_i *be the transfer received by person* i. *All three neighbors are better off by* $20 + 8 - (49 - 25) = 4$ *at the efficient allocation. Transfers* (T_D, T_J, T_M) *obey:* $T_i \in [-4, 8]$ *for all* i *and* $T_D + T_J + T_M = 0$.

(c) If Coasian bargaining is impossible, what snow removal subsidy achieves the socially efficient outcome?

Solution: With a proportional subsidy s *on snow removal, the FOC for a neighbor is* $10 + s = 2x$. *So* $x = 7$ *for the efficient outcome requires* $s = 4$.

5) **Market Power.** OPEC is an oil cartel, but does not control the entire market. Residual suppliers from the rest of the world fill in *after OPEC sets their quantity.* [20]

Assume an inverse world oil demand curve $P_D(Q) = A - BQ$ and a competitive residual supply with marginal cost $c > 0$, so that $Q_R(p) = p/c$. OPEC has costs $C(Q_0) = kQ_0$ for production Q_0, where $0 < k < Ac/(B + c)$.

Find the world oil price, OPEC's quantity supplied, and the residual oil supply. Show that the market price increases in k and c.

Solution: By backward induction, if OPEC supplies Q_0, *then market quantity is* Q *iff the residual quantity is* $Q_R = Q - Q_0$. *The market clears if* $P_D(Q) = P_S(Q)$:

$$Q = Q_R + Q_0 = p/c + Q_0 = P_D(Q)/c + Q_0 = (A - BQ)/c + Q_0$$

Then the market quantity is a function $\mathcal{Q}(Q_0) = (A + cQ_0)/(B + c)$ *of OPEC's quantity. The market price* $P(Q_0) \equiv P_D(\mathcal{Q}(Q_0)) = A - B(A + cQ_0)/(B + c)$ *falls in OPEC's output* Q_0. *OPEC's profit maximization is*

$$\max_{Q_0} Q_0 P(Q_0) - kQ_0 = Q_0[A - B(A + cQ_0)/(B + c)] - kQ_0$$

Its first period FOC is then

$$\begin{aligned}
& 0 = [A - B(A + cQ_0)/(B + c)] - Q_0 Bc/(B + c) - k \\
\Rightarrow\quad & A(B + c) - BA - k(B + c) = 2BcQ_0 \\
\Rightarrow\quad & Q_0^* = \frac{Ac - k(B + c)}{2Bc}
\end{aligned}$$

Our initial assumption on OPEC marginal costs $k > 0$ *implies* $Q_0^* > 0$. *[In a special case, with infinite competitive costs* $c = \infty$, *the market quantity is the standard monopoly quantity for linear demand* $\bar{Q}_0 = (A - k)/2B$.*]*

We can now substitute in Q_0^*, *to solve for the world market price of oil:*

$$P(Q_0^*) = A - B\frac{A + cQ_0^*}{B + c} = A - B\frac{A + \frac{Ac - k(B+c)}{2B}}{B + c} = \frac{Ac}{2(B + c)} + \frac{k}{2}$$

Given market price $p = P(Q_0^*)$, *the residual quantity is* $Q_R(p) = p/c = \frac{A}{2(B+c)} + \frac{k}{2c}$. *Intuitively, higher cartel costs* k *increases residual quantity. [In a special case, the maximal* $k = Ac/(B + c)$ *means* $Q_R(p) = \frac{A}{B+c}$, *and we reduce to pure competition.]*

6) **The Luck of the Iris.** Iris and Joe trade Arrow-Debreu securities in period 0. One of two states happens in period 1: ω_1 with chance $p \in (0,1)$ and ω_2 with chance $1-p$. Each is endowed with w units of the consumption good x. Iris has utility $u(x) = \log(x)$, while Joe has state dependent preferences: utility $u_1(x) = \log(x)$ in state ω_1 and $u_2(x) = a\log(x)$ in state ω_2, with $a > 1$. How does the equilibrium price of consumption in ω_2 relative to ω_1 change as a or p increases? [15]

Solution: Let q be the price of consumption in state 2 relative to state 1. Each individual has wealth $(1+q)w$ that they split between the two states.

Iris solves:

$$\max_{c_1^I, c_2^I} p \cdot \log(c_1^I) + (1-p) \cdot \log(c_2^I) \ \ s.t. \ \ c_1^I + qc_2^I \leq (1+q)w$$

His FOC is:

$$\frac{pq}{c_1^I} = \frac{1-p}{c_2^I} \Rightarrow c_2^I = \frac{1-p}{pq} c_1^I$$

Substituting into our budget constraint, $c_1^I + \frac{1-p}{p} c_1^I = (1+q)w \Rightarrow c_1^I = p(1+q)w$.

Joe solves:

$$\max_{c_1^J, c_2^J} p \cdot \log(c_1^J) + a(1-p) \cdot \log(c_2^J) \ \ s.t. \ \ c_1^J + qc_2^J \leq (1+q)w$$

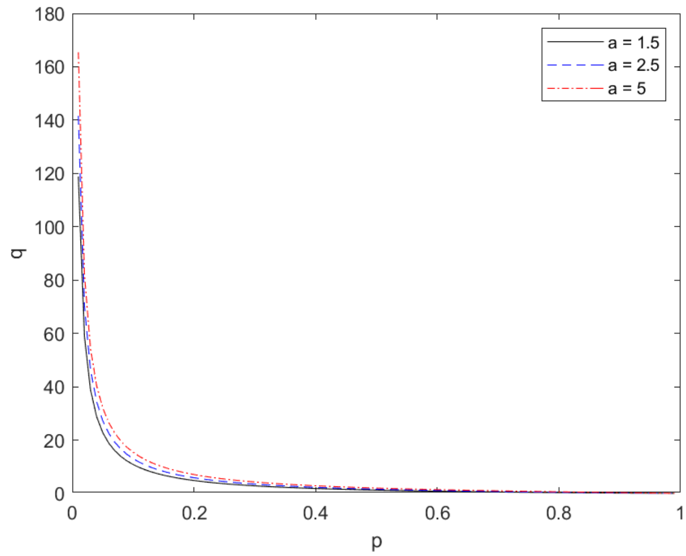
His FOC is $c_2^J = \frac{a(1-p)}{pq} c_1^J$*. Plugging into our budget constraint,*

$$c_1^J + \frac{a(1-p)}{p} c_1^J = (1+q)w \Rightarrow c_1^J = \frac{p(1+q)}{p + a(1-p)} w$$

By Walras Law, we need only clear the ω_1 market:

$$p(1+q)w + \frac{p(1+q)}{p+a(1-p)} w = 2w \Rightarrow q = \frac{(1-p)(p+a(2-p))}{p(1+p+a(1-p))}$$

As is intuitive, the price of the consumption good in state ω_2 falls in p, and rises in a. The price of consumption in state ω_2, as a function of p across values of a:



7) A unit mass of workers with labor disutility distributed uniformly on $[1,2]$ supplies labor L_i^s at wage w and earns profit π from owning the firm. Assume all agents own an equal share in the representative firm with production technology $Y = f(L) = L^\alpha$ for $\alpha \in (0,1)$. Agent i with disutility ϕ_i has utility $u(c_i, L_i) = \log(c_i) - \phi_i L_i$. [15]

(a) What is the firm's labor demand and profits as a function of the wage?
Solution: The firm solves:

$$\pi = \max_L L^\alpha - wL$$

It FOC is:

$$\alpha L^{\alpha-1} = w \Rightarrow L = \left(\frac{\alpha}{w}\right)^{1/(1-\alpha)}$$

Hence, profits are:

$$\pi = (1-\alpha)\left(\frac{\alpha}{w}\right)^{\alpha/(1-\alpha)}$$

(b) What is consumer i's consumption and labor in terms of the wage and profits?
Solution: The consumer solves:

$$\max_{c_i, L_i^s} \log(c_i) - \phi_i L_i^s \ \text{s.t.}\ c_i \leq wL_i^s + \pi$$

The FOC is $c_i = w/\phi_i$ — independent of π. Labor supply is then $L_i^s = \frac{1}{\phi_i} - \frac{\pi}{w}$

(c) Find the market wage and production Y. What is the elasticity of Y in α?
Solution: We clear the consumption good market.

$$\left(\frac{\alpha}{w}\right)^{\alpha/(1-\alpha)} = \int_1^2 \frac{w}{\phi_i} d\phi_i = \log(2) - \log(1) = \log(2)$$

So $w = \alpha(\log(2))^{(1-\alpha)/\alpha}$. Thus, $Y = \left(\frac{\alpha}{w}\right)^{\alpha/(1-\alpha)} = \frac{1}{\log(2)}$ is independent of α — zero elasticity.

8) **Thinker Problem About Markets.** Assume an exchange markets for a good in two cities A and B. Competitive prices are $p_A < p_B$ and quantities are q_A, q_B. Then the markets merge. How does the new competitive price compare to p_A and p_B? How does the new competitive quantity compare to $q_A + q_B$? Is total trade surplus higher or lower after the merger? Please rigorously argue all claims. [20]

Hint: Produce very simple examples where quantity traded rises or falls.

Solution: Supply and demand balance in market B, and supply exceeds demand in market A at $p^* = p_B$*, and demand exceeds supply at* $p^* = p_A$ *in market B, and supply balances demand in market A at* $p^* = p_A$*. Altogether, the merged market has a price in* (p_A, p_B)*.*

Surplus is higher in the merged market with the one price, rather than say two prices p_A, p_B *— by the first welfare theorem. This does not necessarily imply that quantity rises, however. Consider two simple examples:*

***Quantity falls**: Assume one potential buyer and seller in markets A and B. The buyer in market A values the good at* 7*, while the seller incurs an opportunity cost of* 5*. In market B, the buyer has a value of* 4 *while the seller has opportunity cost* 1*. Two goods are traded in our separated markets, but only one good is traded in the merged market. The gains from trade increase from* $2 + 3 = 5$ *to* 6 *in our merged market, despite a lower quantity traded.*

***Quantity rises**: Let there again be one potential buyer and seller in markets A and B. The buyer in market A values the good at* 2*, while the seller incurs an opportunity cost of* 3*. In market B, the buyer has a value of* 4 *while the seller has opportunity cost* 5*. No goods are traded when our markets are isolated, but in the merged market one trade occurs. Thus, quantity can also rise when markets are merged.*