An Economic Theory Masterclass

Part II: Competitive Markets in Partial Equilibrium

Lones Smith

February 10, 2025

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Paul Samuelson Produced this Economic Idea

• And not Chadwick Boseman; murder is legal in self-defense



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Supply and Demand

- Assume a competitive price-taking environment
- Double auctions: just an extensive margin (in or out) for all trades
 - WTP (willingness to pay) and WTA (willingness to accept)
- Supply & demand curves will also reflect intensive margins
- usually upward sloping supply curve
- usually downward sloping demand curve
 - very negative income effects \Rightarrow demand rises in price
 - addictive behavior \Rightarrow WTP rises with quantity (oh no, drugs)
- These two curves answer out-of-equilibrium hypothetical "what if" questions: what would the supply and demand be at any other price?
- By parsing our logic into supply and demand, we can compartmentalize our analysis, and make clearer predictions
 - Supply and Demand: "Father Guido Sarducci's 5 Minute University"

Ours "Static" Models are Really Steady States

- Supply quantity $S = Q^S$ and inverse supply price P
- Demand quantity $D = Q^D$ and inverse demand price P
- The model need not be static. Everything could be steady-state!
 - Supply and demand could be flows (units are per week, or per day)
 - Life is all about dynamics: Heraclitus Panta Rhei
 - "All entities move and nothing remains still"
 - "No man ever steps in the same river twice"



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Definitions

Before

taking

theory



After taking economic theory

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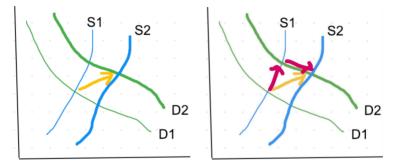
Definitions

Rear View Mirror on Matching (TU)



- Becker Marriage: PAM/NAM⇔SPM/SBM (extremes!)
- $\bullet\,$ Second Welfare Theorem and profit maximization by middlemen \Rightarrow wages
- Double auctions specialized Shapley-Shubik to identical units
- $\bullet\,$ Trade surplus is SBM $\Rightarrow\,$ NAM matching in a double auction
- Law of one price is efficient and maximizes gains from trade
- The crossing of supply and demand determines quantity:
 - reverse ordered valuations 40, 36, 32, 28, 24, 20, 16, 12, 8, 4
 - \bullet Ordered costs: 3, 9, 15, 21, 27, 33, 39, 45, 51, 57
 - Market quantity $k^* = 4$ and price: $\max(24, 21) = 24 \le p^* \le 27 = \min(28, 27)$
 - How big can a unit tax t > 0 or subsidy s > 0 be and still have $k^* = 4$?
- Nonmarket mechanisms (military drafts, organ assignment, gifts) do not maximize gains from trade
- The course now shifts to markets without pairwise trades
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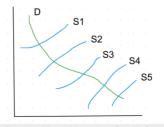
Comparative Statics aka Comparison of Steady States Analysis



- Comparative statics are a peasant's comparative dynamics
- Intuitively, monotone dynamics from one steady-state to the next \Rightarrow comparing the two static situations is informative of dynamics
- What if demand shifts quickly, but supply shifts slowly?

Identification of Supply and Demand Curves

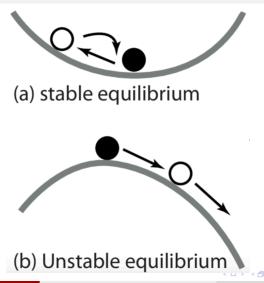
- Price and quantity reflect both supply and demand.
- If you wanted to "identify" the demand curve, you find something that just shifts supply and leaves demand invariant.
- Ragnar Frisch (1933) first highlighted the identification problem first winner of Economics Nobel prize (1969)
- With enough variation in supply, we can identify the demand.
- Likewise, variation in demand but not supply would allow one to pin down the supply curve.



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Stability





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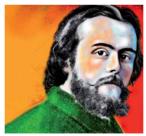
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Stability

Stability: Does Competitive Equilibrium Happen?

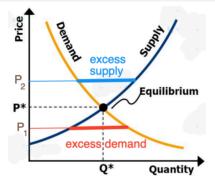
- Why does market equilibrium arise?
- adjustment *tatonnement process* check Google translate :)
- Walrasian price stability (Elements of Pure Economics, 1874)
 - price adjustment process of fictional double auctioneer
 - \Rightarrow change in the price shares the sign of *net demand* $Q^{D}(P) Q^{S}(P)$.



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Walrasian Stability in Standard Case: Demand \searrow and Supply \nearrow



- Dvnamic stories
 - Search by people who engage in pairwise bargaining over prices
 - forward-looking optimization about willingness to accept
 - During the adjustment, the short side of the market fixes quantity.
 - Demanders won't demand more than they want at that price.
 - Suppliers won't sell more than they are willing at that price.

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Stability

Applied Detour: The Market "Learns"

- The market is the ultimate in artificial intelligence
- Groups of individuals might screw up but the market learns
- Even capuchin monkeys can learn supply and demand Science News from research organizations

Monkeys Follow Economic Rules Of Supply And Demand

- Date: September 8, 2009
- Source: NWO (Netherlands Organization for Scientific Research)
- Summarv: A monkey that has acquired the sole power to hand out apples is generously rewarded with arooming sessions by the other monkeys in its group. But as soon as another monkey can hand out apples as well, the market value of the first monkey is halved. The monkeys therefore unerringly obey the law of supply and demand.



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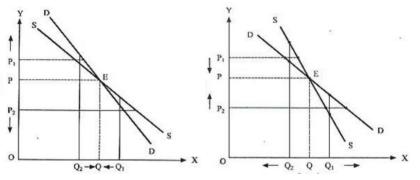
Game Theory Detour: Financial Crisis of 2008

- When markets do not learn, we are stunned
- How could the price not clear the market?
- Money is not the medium of exchange for big trades
- Buying with margin (taking a loan)
 - \Rightarrow your collateral falls in value when prices fall
 - \Rightarrow demand might fall when price falls
 - \Rightarrow upward sloping demand and supply
 - \Rightarrow game of strategic complements
 - \Rightarrow multiple equilibria
- Advanced Theory Topic: Games of Strategic Complements

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Image: A mathematical states and a mathem

Walrasian Stability with Downward-sloping Demand and Supply

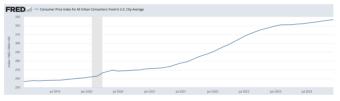


- Supply steeper than demand \Rightarrow Walrasian stable
- Demand steeper than supply \Rightarrow Walrasian unstable
- So Walrasian stability holds iff $Q_P^S(P) > Q_P^D(P)$
 - ...formulated using direct and not inverse supply & demand curves!
- Not Even A Thinker Q: What if supply and demand slope up?

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Supply and Demand: Why is Food Inflation > Average Inflation

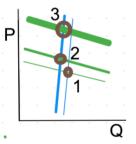
- Assume COVID Stimulus (\$2.97 in 2020 and \$1.97 in 2021) Raised Demand
- Food in Cities (24.7% Inflation 2020-24)



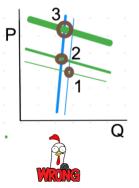
• All Urban Goods (19.3% Inflation 2020-24)



- Consider the facts of the oil or gasoline market
 - Huge price volatility
 - Minimal quantity volatility
 - Small change in fundamentals (i.e. small shift in supply and demand)

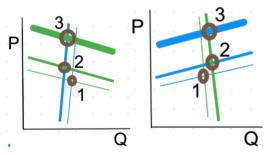


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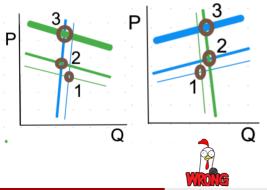


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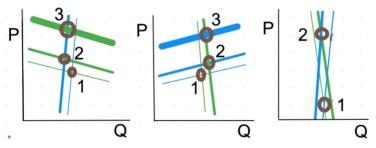


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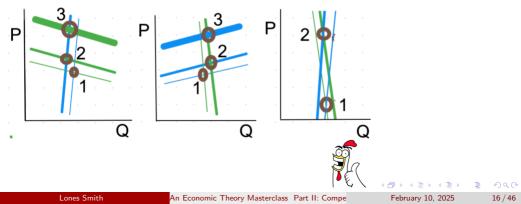


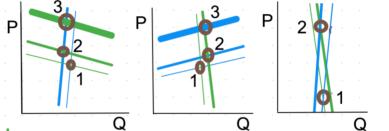
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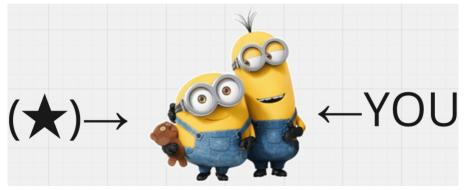




- Small fundamentals shifts cause large proportionate price changes iff both supply and demand are nearly vertical.
- Nearly vertical supply or demand \Rightarrow low quantity volatility
- Small fundamentals changes can lead to large quantity changes iff supply and demand are both nearly horizontal.
- Nearly horizontal supply or demand \Rightarrow low price volatility

Big Picture Comparative Statics Insight

- In markets, the market-clearing condition (\bigstar) is your friend.
 - $\rightarrow~$ If you can, differentiate it
 - $\rightarrow\,$ Trick: simplify its derivative by plugging (\bigstar) back into it



Deja Vu: Flash Elasticities Review of Economics 711

• For small price changes:

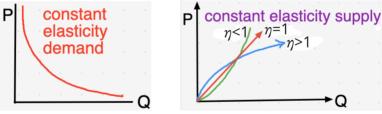
(upper case)
$$\mathcal{E}(Q, P) = \frac{dQ}{dP} \frac{P}{Q} = \frac{d \log Q}{d \log P} \approx \frac{\% change \ quantity}{\% change price}$$

- $\Rightarrow~$ Coefficients in log regressions are elasticities
 - Elasticity is a ratio of proportionate changes \Rightarrow unit-free!
 - More elastic supply or demand \Rightarrow quantity changes more if price falls
- The long run has fewer constraints than the short run
- Le Chatelier's Principle: The absolute change of any choice variable is higher in the longrun than shortrun. ⇒ |long run elasticity| > |short run elasticity|



Constant Elasticity Supply and Demand Curves

- Supply elasticity $\eta = Q_P^S(P)P/Q^S > 0$
- Demand elasticity $\varepsilon = Q_P^D(P)P/Q^D < 0$
 - PS Demand elasticity is spoken of in absolute terms!
- On this slide, let's assume ε and η are constant
- Integrating $dQ^D/Q^D = \varepsilon dP/P \Rightarrow \log Q = \varepsilon \log P \varepsilon \log K \Rightarrow Q = (1/K^{\varepsilon})P^{\varepsilon}$.
- Hyperbolic downward sloping curves $\varepsilon < 0$: $P = KQ^{1/\varepsilon}$



• Geometric upward sloping supply curves ($\eta > 0$) are linear if $\eta = 1$ • Supply is elastic if $\eta > 1$ and demand is elastic if $|\varepsilon| > 1$

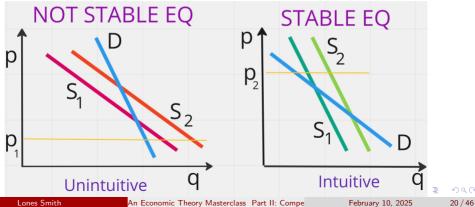
Quantity changes proportionately more than price Lones Smith

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Samuelson's Correspondence Principle (1941)

- Comparative statics are "intuitive" if the equilibrium is stable: price falls if supply rises, or demand falls
- Standard case: increasing supply and decreasing demand
- More subtle cases: direct supply curve is steeper than demand



Tax Irrelevance Theorem

- Double auctions: No effect of small tax! So we assume continuous quantity!
- Tariff or sales or *ad valorem* tax: $P^{D}(Q) = P^{S}(Q) + \tau P^{S}(Q)$
- Specific tax τ : $P^{D}(Q) = P^{S}(Q) + \tau$ (also, an excise tax)
 - Wisconsin specific tax examples
 - Gas tax: state 32.9¢ and federal 18.4¢ per gallon (EV's pay an annual fee)
 - Beer: 6/gallon and wine: 25/gallon and liquor: 3.25/gallon
 - Also exists for cigarettes
- Specific tax is easier to analyze: parallel demand / supply shift

Theorem (Tax Irrelevance Theorem)

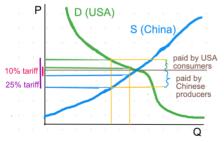
Regardless of whether demand or supply pays a specific tax, the demand and supply prices, market quantity, and efficiency loss are the same.

- $\bullet~$ USA: A sales tax is paid by demanders \Rightarrow down-shift in demand
- Most of world: VAT (hidden tax) is paid by suppliers ⇒ up-shift in supply, since the marginal cost of sellers is higher by the tax

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Incidence Incidence of a Tax or Tariff

- $\bullet\,$ 2018, Trump put a 10% then 25% tariff on Chinese imports. Today 10% more
- \Rightarrow wedge between supply and demand prices: $P_D > P_S$.
 - WRONG: "China is paying us billions of dollars in tariffs." Trump
 - Incidence: Who pays the tariff or tax, accounting for the price change?
 - Idea: The more elastic is demand, the less of the tariff buyers pay.
 - Idea: The more elastic is supply, the less of the tariff suppliers pay.



👂 Why does POTUS have the Constitution right to start a tariff — i.e. a tax on importers? Only Congress can levy taxes. 💈 🗠 🔍 🔍

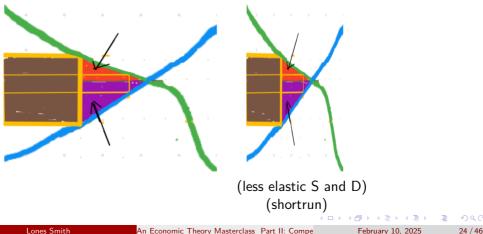
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Deadweight Loss of Tax

- Lost gains from trade = lost consumer + producer surplus
- Assume tariff revenue is socially neutral: gain to government balances loss to producers or consumers
- ⇒ deadweight loss (excess burden) of tariff is red + purple
 Deadweight loss -- consumer surplus side
 Tariff
 Revenue
 Deadweight loss -- producer surplus side

Changes in the Deadweight Loss of Tax

• The deadweight loss of a tariff increases in the quantity reduction, larger with more elastic demand or supply



Analytical Comparative Statics (Smooth Supply or Demand)

- Add a shift parameter to supply $Q^{S}(P|\beta)$, with $Q_{\beta}^{S}(P|\beta) > 0$
- Competitive equilibrium price & quantity solve: $Q^{D}(P) = Q^{S}(P|\beta)$
- Implicitly differentiate equilibrium identity in β , with $P(\beta)$ a function:

$$\frac{dP}{d\beta} = \frac{-Q_{\beta}^{S}(P|\beta)}{Q_{P}^{S}(P|\beta) - Q_{P}^{D}(P)} \qquad (\bigstar)$$

- \Rightarrow Price falls when supply rises, provided stable: $Q_P^S(P,\beta) > Q_P^D(P)$
 - Multiply (\bigstar) by $(\beta/P) = (\beta/Q)/(P/Q)$. Then the equilibrium price elasticity is $\mathcal{E}(P|\beta) = \frac{dP\beta}{dP} - \frac{-\mathcal{E}(Q^S,\beta)}{dP}$

$$\mathcal{E}(P|\beta) \equiv \frac{d\eta}{d\beta} \frac{\beta}{P} = \frac{-\mathcal{E}(\mathbf{Q}^{*}, \beta)}{\eta - \varepsilon}$$

• Add a shift parameter to demand $Q^D(P|\alpha)$, with $Q^D_{\alpha}(P|\alpha) > 0$.

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• Price rises if demand increases, given a stable equilibrium. Indeed:

$$\frac{dP}{d\alpha} = \frac{Q^{D}_{\alpha}(P|\alpha)}{Q^{S}_{P}(P|\beta) - Q^{D}_{P}(P|\alpha)} = \frac{\mathcal{E}(Q^{D}|\alpha)}{\eta - \varepsilon} \leftarrow \text{Homework}$$
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Tax Incidence

Elasticities and Tax Incidence: Who pays the tax?

- A small tax has no effect in a double auction. So shift to continuous quantity.
- The more inelastic side of the market pays more of a tax and benefits more from a subsidy, but how much more?
 - Demand elasticity $\varepsilon = D'(P)(P/D) < 0$
 - Supply elasticity n = (dS/dP)(P/S) > 0

Theorem (Tax Incidence Theorem)

The share of a small tax τ paid by demand is $\frac{\eta}{n-\varepsilon}$, and by supply is $\frac{-\varepsilon}{n-\varepsilon}$.

- *Proof*: By Tax Irrelevance Theorem, impose the tax τ on demand.
- Differentiate $D(P(\tau) + \tau) \equiv S(P(\tau))$, where $P(\tau)$ is supply price
- Hence, $D'(P(\tau) + \tau)(P'(\tau) + 1) = S'(P)P'(\tau)$
- Supply price slope in the tax:

$$\Rightarrow \qquad P'(\tau) = \frac{D'(P(\tau) + \tau)}{S'(P) - D'(P(\tau) + \tau)} \approx \frac{\varepsilon}{\eta - \varepsilon} \in (-1, 0)$$

• Finally, demand price rises with slope $P'(\tau) + 1 \approx \eta/(\eta - \varepsilon) \in (0, 1)$

Deadweight Loss for Small Taxes

• Since $\varepsilon = D'(P)(P/D)$, the quantity demanded changes by

$$dQ = \epsilon \frac{QdD}{D} \approx \epsilon \left(\frac{\eta}{\eta - \epsilon}\right) \tau \left(\frac{Q}{P}\right) = \left(\frac{1}{\frac{1}{\epsilon} - \frac{1}{\eta}}\right) \tau \left(\frac{D}{P}\right)$$

- Check the units in this formula! Answer: $(\$/q) \cdot \frac{q}{\$/a} = q$ (the unit of Q)
- Deadweight loss: Lost gains from trade = lost CS + PS
- Hence, the deadweight loss is the area of the standard triangle:

$$rac{1}{2}(dQ)(dP^D-dP^S)=rac{1}{2}(dQ) aupprox \left(rac{1}{rac{1}{\epsilon}-rac{1}{n}}
ight)\left(rac{Q}{2P}
ight) au^2$$

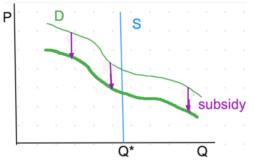
• Check the units in this formula! Answer: $\frac{q}{\$/q} \cdot (\$/q)^2 = \$$

- Thinker: What about Quantity Taxes?
 - Feudal system: Give a tithe of crops to the church!
 - Tithe τ : $P^D(Q) = P^S(Q + \tau)$

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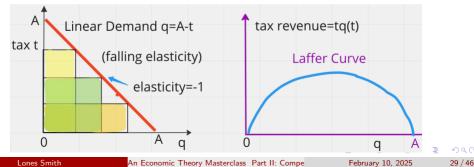
Political Economy of Taxes: Tax or Subsidy Incidence



- Tax or subsidy incidence invariably explains who pushes for it
- In 2009, Michigan ended the Promise Scholarship program, giving 96,000 in-state students up to \$4,000 for college
 - Can't \uparrow shift supply curve \Rightarrow shift demand (Tax Irrelevance Theorem)
 - Who fought to keep the subsidy? Colleges! (Tax Incidence Theorem)
- Take our message for governments: taxing inelastic supply is efficient

Demand Elasticity and the Laffer Curve for Total Revenue

- Tax revenue tq(t) is rising / falling when $tq'(t) + q(t) \ge 0$ iff $\varepsilon \ge -1$
- If tax revenue peaks at an intermediate quantity, then this rules out a constant elasticity demand
- Linear demand curves have falling elasticities $|\varepsilon| = \left| \frac{dq \ p}{dp \ q} \right| = p/q$
- Tax revenue is maximized (in example midway, as slope is minus one)



Art Laffer's 1974 Back of the Envelope Explanation to Rumsfield

It you take a product loss results is substitute a " more a " We been thring work with and image And substituting non-with, lessure and un-employment. The consecutiones are obvious Probibitive Roomse Normal Range To Der Rousteld. To Der Rousteld. 4 and The Cartas 4 Real J 9/ 13/ 94 Gard B. Affe

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Rear View Mirror on Competitive Supply and Demand





- Demand curve fall & supply curves rise \Leftrightarrow heterogeneity & convexity
- Both P and Q change given shocks Q more with greater elasticity
- Stability \Leftrightarrow supply and demand elasticities $\eta > \varepsilon$
- $\bullet~$ Correspondence Principle: stability $\Rightarrow~$ intuitive comparative statics
- Less elastic side of market pays more of a tax (political economy 101)
 - Laffer curve. PS Optimal taxation says tax more elastic goods less (${\scriptstyle \blacksquare}$)

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Public Finance: the Ramsey Inverse Elasticity Tax Rule

- Social planners hate deadweight losses
 - \Rightarrow Optimal taxes minimize deadweight losses for any given revenue
- Tax revenue falls when the tax rises if the demand is elastic:

$$[D(P+\tau)\tau]' = D(P+\tau) + D'(P+\tau)\tau = D(P+\tau)[1+\varepsilon\frac{\tau}{P+\tau}]$$

 \Rightarrow never tax an elastically demanded good

- Ramsey (1927): Minimize the social cost of raising revenue Rmax $V(p + \tau, I)$ s.t. $\tau \cdot x(p + \tau, I) \ge R$
 - where V(p, I) is the indirect utility function for prices p and income I
 - Cool! This long predates the 1950 invention of Kuhn Tucker analysis!!
- Ramsey inverse elasticity rule: "taxes should be proportional to the sum of the reciprocals of its supply and demand elasticities"
- → governments shouldn't tax elastically demanded goods or supplied goods

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Optimal Taxation Theory Explains Real World Taxes

- Ramsey's basic insight is intuitively understood by governments
- They know to tax inelastically supplied resources:
 - Oil taxes, mineral taxes
 - existence tax: poll tax (head tax) in Britain (fertility impact?)
 - wealth taxes are usually real estate, or at death taxes
 - millionaire tax? billionaire tax?
- More rationality ++++ more elastic response
 - Example: Does income reflect effort, ability, luck or networks?
 - Tax luck or ability or networks inelastically supplied. Politically:
 - left wing thinks earnings reflect luck & networks more, right wing effort
 - $\bullet~$ left wing understates elasticities $\Rightarrow~$ higher peak of Laffer curve
- Funny example of a tax fail:
 - 2008, Maryland "millionaire's tax" of 6.25% tax on income > \$1M
 - 30% drop in millionaire's taxpayers and 22% drop in declared income.
 - $\Rightarrow\,$ income taxes from this group fell by \$257 million
 - Tax ended in 2010

Trump's Tariff Threats Against Canada & Mexico

- Trump threatened Canada with 10% oil tariffs, 25% tariffs on other goods
- Canada responded to 25% tariff on \$150B of goods
- Open Thinker: How should Canada respond, to cause as much harm to the US, and little harm to Canada?
- China responded also with export limits on rare minerals
- quantity space!





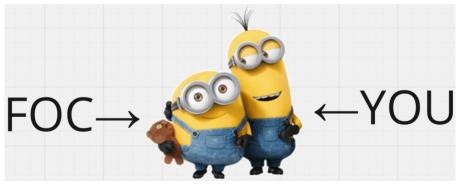
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Big Picture Optimization Insight

- In *differentiable optimizations*, the FOC is your friend.
 - $\rightarrow\,$ If you can, differentiate it
 - $\rightarrow\,$ Trick: simplify its derivative by plugging the FOC back into it



• In nondifferentiable optimizations, use monotone comparative statics

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An Economic Theory Masterclass Part II: Compe

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Stability is the Social Planner's SOC

- Lemma: When supply and demand are decreasing, surplus is $\int_{p}^{\infty} D(z) S(z) dz$
 - Proof: Integrate surplus $\int_{p}^{\infty}P^{D}(q)-P^{S}(q)dq$ by parts
- Maximize surplus $\int_{p}^{\infty} D(z) S(z) dz$ at competitive equilibrium
- Surplus FOC $D(p) S(p|\beta) = 0$ holds at equilibrium
- Differentiate the FOC or equivalently market clearing condition

• Use the SOC
$$D_p(p) - S_p(p|eta) \leq 0$$

- Stability \Leftrightarrow SOC of planner!
- \Rightarrow Stable equilibrium is a local welfare max

$$p'(eta) = rac{-S_eta(P|eta)}{S_p(p|eta) - D_p(p)} \propto -S_eta(p|eta)$$

- ullet This applies at both stable equilibria \nearrow
- Only one equilibrium is globally optimal.



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Supply / Demand Curves: Intensive and Extensive Margins

- All gains from trade in the double auction producer plus consumer surplus — reflect heterogeneity, and an extensive margin (in or out)
- In most markets, optimizers also adjust an intensive margin
- Intensive margins will reflect local optimality: FOC and SOC
- Extensive margins will reflect global optimality: entry and exit

Deja Vu: Flash Cost Function Review of Economics 711

- Escapable costs can be avoided vs. sunk (inescapable) costs
 - "Sunk costs are sunk": they cannot possibly affect dynamically rational behavior, and should be ignored
 - = essence of dynamic programming
- A fixed cost is invariant to the quantity.
 - It can be sunk or escapable
- A variable cost has an intensive margin
 - So variable costs are escapable (just vary them down to zero)
 - Marginal costs are the derivative of variable costs
 - Average costs are fixed plus variable costs divided by quantity
- Optimization Big Picture
 - All firms intensive margin: marginal costs equals price
 - All firms extensive margin (no exit) Average costs \leq price \Leftrightarrow
 - Marginal firm: Average costs = price \Leftrightarrow extensive margin (no entry)

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Deja Vu: Short, Medium, Long Runs Review of Economics 711

- As the run increases, there are more choice margins, and so inescapable costs ~> escapable (e.g., rental contracts end).
- Short run
 - I fixed costs are inescapable; cost function is just variable costs
 - Insufficient time for entry; reducing output to zero
 - Ukraine consumes entire UK supply of artillery every 8 days!
- Long run
 - I All costs are escapable, and so are included in the cost function
 - \Rightarrow Costs are higher in the long run than short and medium runs
 - Ø firms enter if there are profits to be made and otherwise exit
 - John Maynard Keynes: "In the long run we are all dead"
 - Naturally, Keynes developed a short run theory

• "Medium run"

- more decision margins available
 - \Rightarrow more costs escapable than in short run
 - \Rightarrow fewer costs escapable than in long run
- Time Magazine Cover $12/31/1965 \longrightarrow$

Lones Smith



Long Run Supply with Homogeneous Firms and Intensive Supply

- Goal: show how intensive and extensive margins interact
- We explore an illustrative extended example, focusing on supply!
- Industry supply curve locus (Q, P)
 - Fixing *P*, existing firms *i* in the short run, or all potential firms in the long run — profitably produce q_i , and $Q = q_1 + \cdots + q_n$ in real world with *n* firms
 - Continuum model $Q = \int_0^m q_i di$ with mass *m* firms ("all models are wrong")
 - Price-taking behavior is incredible with few firms
- Cost functions $C(q) = 1 + q^2$ (fixed cost 1 & variable cost q^2)
- Optimal production: $C'(q) = P \Rightarrow$ output $q^* = P/2$.
- Long Run
 - No firm wishes to enter or exit, with all costs escapable: P = C(q)/q

$$2q^* = C'(q^*) = P = C(q^*)/q^* = rac{1}{q^*} + q^* \Rightarrow 2q^* = rac{1}{q^*} + q^* \Rightarrow q^* = 1 \Rightarrow P = 2$$

- \Rightarrow The long run inverse supply curve is P = 2.
 - Every firm earns zero profits in the long run

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Short Run Supply with Homogeneous Firms and Intensive Supply

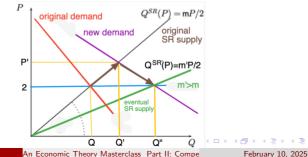
- Short run: each firm still produces $C'(q) = P \Rightarrow$ output $q^* = P/2$
 - This intensive margin effect firms sell more with a higher price was absent with double auctions
- Fix the mass m of firms $\Rightarrow Q_m^{SR}(P) = mq = mP/2$ (1) $Q^{SR}(P) = \mathsf{m}P/2$ longrun supply 2 m/2

All firms earns positive profits: C_{SR}(q) = (q*)² ⇒ AC = q* < P
 The short run supply curve rises simply due to cost convexity, z →

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Short Run and Long Run Response to a Demand Increase

- Short run
 - Every firm produces more (along its marginal cost curve)
 - The price increases to P'>2 and the quantity to $Q'=Q^{SR}(P')>Q$
 - Quasi-rents: temporary positive profits during adjustment (AC < P)
- Long run (after enough time passes so that entry occurs)
 - Firm mass rises to m' > m so that short run supply allows P = 2
 - \Rightarrow quantity rises to Q'' > Q'
 - Entry \Rightarrow long run supply is more elastic (Le Chetalier's Principle)



Supply with Heterogeneous Firms and Intensive Supply

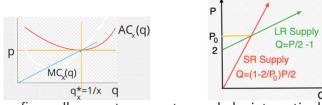
- Firm with index x has costs $C_x(q) = 1 + x^2 q^2$
- Assume the index x has a unit mass density on $[1,\infty)$
- Higher index firms produce less output q_x when positive

• Firm x supplies $2x^2q_x = MC_x = P \Rightarrow$ supply $q_x(P) = P/(2x^2)$ (low)

- Short run: No one shuts down, since the price exceeds non-sunk costs: $AC_x(q) = x^2q_x < 2x^2q_x = MC_x(q) = P$
- Long run
 - The fixed cost 1 is escapable, and included in costs
 - \Rightarrow U-shaped $AC_x(q) = 1/q_x + x^2q_x = 2x^2/P + P/2 \le P$ for all firms $x \le \frac{1}{2}P$
 - \Rightarrow minimum efficient scale of firm x is $q_x^* = 1/x < 1$.
 - \Rightarrow The minimum average cost is $AC_x(q^*_x) = 1/q_x + x^2q^*_x = 2x \ge 2$
 - Marginal firm earns 0 profits at min AC: $P = AC_x(q_x^*) = 2x$
 - Why? The min AC is the most efficient a firm can be!
 - \Rightarrow Marginal firm is $x(P) = \frac{1}{2}P$
 - Price \geq 2: must pay for minimum average costs
- Thinker: Find long run supply for costs $C_x(q) = x + q^2$.



Long Run vs. Short Run Supply with Heterogeneous Firms



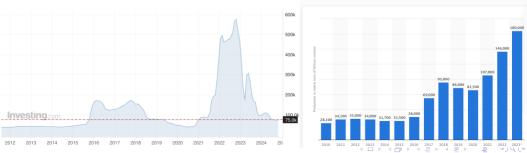
• Continuous firms allows us to compute supply by integration!

- Long run supply is all supply (l) by inframarginal firms $x \le x(P)$: $Q_S^{LR}(P) = \int_1^{x(P)} q_x(P) dx = \int_1^{P/2} P/(2x^2) dx = [P/2][-x^{-1}]\Big|_1^{P/2} = \frac{1}{2}P-1$
 - This integral [or "mass" or "measure"] is well-defined for $P \ge 2$.
 - The supply curve now rises due to cost convexity and heterogeneity
 - Market supply is more elastic than firm supply
 - Short run supply starting at a price P_0 , i.e. with marginal seller $x(P_0)$:

$$Q_{S}^{SR}(P|P_{0}) = \int_{1}^{x(P_{0})} P/(2x^{2}) dx = [P/2][-x^{-1}]|_{1}^{x(P_{0})} = (P/2)[1-2/P_{0}]|_{1}^{x(P_{0})} = (P/2)[1-2/P_{0}]|_{1}^{x(P_{0})$$

Application: Natural Resources Have Volatile Prices

- Natural Resources have inelastic supply in the short run, until a new well or mine is dug.
- But for lithium price and quantity are volatile.
- New mines are opened, raising fixed costs, lowering marginal costs



Mine production of lithium worldwide from 2010 to 2023

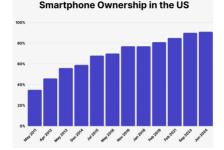
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(in metric tons of lithium content)

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Extensive and Intensive Margins for Smartphones



- Demand with Heterogeneous Consumers:
 - If supply increases and so price falls, the new consumers like the good less and prior consumers buy more
 - Smart phones: inframarginal consumers buy the fancier phones
- Brainstorm imaginative applications of this model: crime rate surge in 2020; mating markets & AIDS?