An Economic Theory Masterclass

Part X: General Equilibrium with Spatial Competition

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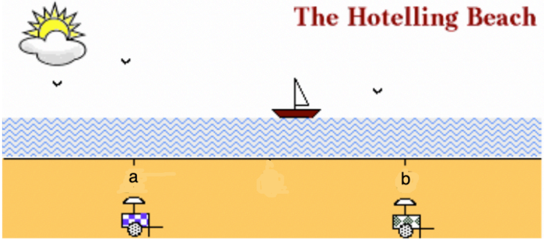
The Hotelling Model

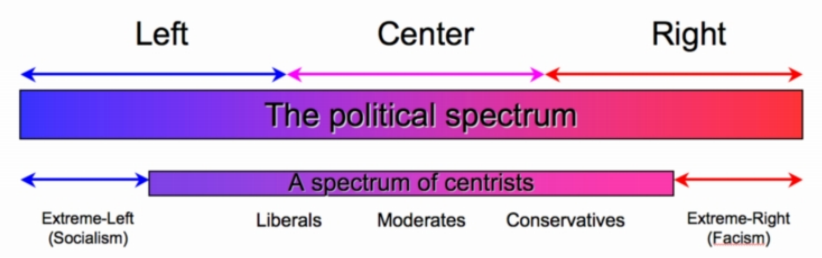
- Harold Hotelling (1929), "Stability in Competition", *EJ*

- Iris and Joe each own lemonade pushcart along a unit beach.
- Iris is located at a and Joe at b, where $0 \leq a \leq b \leq 1$.
- Lemonade is \$2 per glass, *by fiat.*
- Customers are located evenly along beach $[0, 1]$
  - have willingness to pay $v > 1$ for a single cup of lemonade
  - Buyer $x \in [0, 1]$ pays transportation cost $|x - a|$ to walk to a
  - Total sales are independent of where sellers locate (as $v > 1$)
- Given an equal sharing tie break rule if Iris and Joe locate at the same spot, the unique prediction is $a = b = 1/2$.

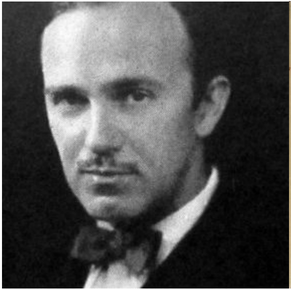
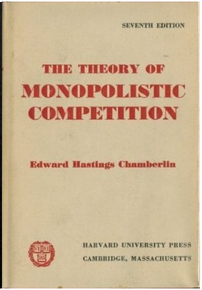
Principle of Minimum Differentiation

- Hotelling predated Nash equilibrium, and is *wrong* if firms set prices
- Highly cited and recommended: d'Aspremont, Gabszewicz and Thisse (1979) famously corrected Hotelling, fifty years later! (on canvas)
- They set up Hotelling as pricing game for any location and show that equilibrium does not exists for closely located firms
- As a location metaphor for a left-right political spectrum, it correctly explains why parties move toward the center
  - If entry is allowed, extreme left and right third parties appear
- Why our current political polarization?
  - I have a dynamic idea (ask me in advanced theory)



Chamberlin's Monopolistic Competition

- Chamberlin, *A Theory of Monopolistic Competition* (1933)

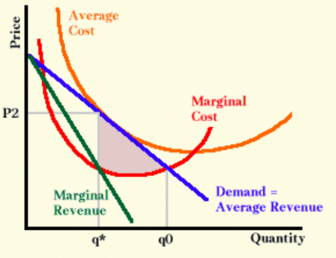
  - Monopolistic: firms to not take prices as given
  - Competitive: $\exists$ free entry and thus zero profits
- Chamberlin allows both price and location competition.
- If two sellers were very close, say near $x = 1/2$, then each seller raises its demand by moving away from the other.
- Why? That lowers the transportation costs for a larger mass of consumers than it raises transportation costs for.
  - Chamberlin coined the term "product differentiation"

Circular Monopolistic Competition

- "Spatial" need not refer to geography
    - Transportation costs may be metaphorical
    - $\Rightarrow$ firm demand curves are falling (steal business from neighbors)
- Firms can freely enter $\Rightarrow$
    - After each entry, demand curves facing all firms shift down
    - marginal firm earns zero profits (e.g. State Street shops)

$\Rightarrow$ Price then exceeds marginal cost when profits vanish at just one quantity q^* (demand curve is tangent to average cost)

- This is really just a model of Bertrand-Nash price competition: since firms have falling demand curves, it is not competitive
- E.g.: economics principles textbooks $\Rightarrow$ Mankiw, Bernanke, Krugman.



Circular Monopolistic Competition in Models

- Hotelling's beach had two ends that were captive markets.
- For many firm applications, we desire a symmetry across firms.
- This suggests using a circle rather than a line segment:

Offline Helpful Detour: Where to Live

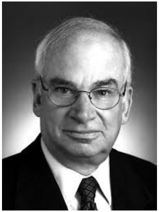
- Consider an in-or-out decision: which city to live in?
- Assume we pick cities for two reasons:
  - money M (wages and cost of living)
  - amenities A (museums, beaches)
- Using the theory, if k's utility is $U_k(M, A) = M + A$, we can impute the **unobserved** factor A from the **observed** factor M
- If consumers k vary by their marginal rate of substitution between M and A, then cities with better M have a lower A
  - Example: If the same caliber worker accepts a wage \$30K less to live in San Francisco than Chicago, then living in SF is arguably worth \$30K more than Chicago
- We now identify simultaneously the equilibrium market clearing values of living in many places

Offline: Where to Live

Flickr/A McLin

How Much Are You Willing to Pay to Live in America's Best Neighborhoods?

Rosen's Competitive Model of Hedonic Pricing



- Multimarket equilibrium in a (figurative) "spatial" market
    - location, product variety (size, power, EV or not of car), etc
- This is an important market design for IO and maybe labor
- Rosen (1974): With small fixed costs, competitive price taking behavior is a better model of product differentiation
- Hedonic prices are the implicit prices of attributes, as revealed by the observed prices of differentiated products.
- *Market-clearing competitive price function of characteristics* $\mathbf{z}$

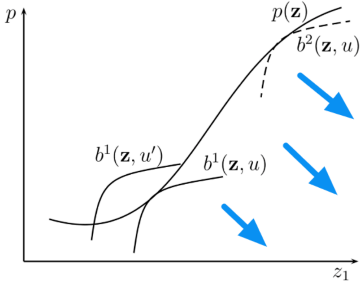
$$p(\mathbf{z}) = p(z_1, \ldots, z_n)$$

The Consumer's Spatial Problem

- Utility $U(x, \mathbf{z})$ depends on money x and $\mathbf{z} = (z_1, \ldots, z_n)$.
  - Competition so far: For every price, consumers optimally pick quantity
  - Here: *For every price function*, consumers pick location & quantity
- The consumer with utility U and money income y solves

$$\max_{(x,\mathbf{z})} U(x, \mathbf{z}) \text{ s.t. } x + p(\mathbf{z}) = y$$

- The bid function $b(\mathbf{z}, \bar{u})$ solves $U(y - b, z_1, \ldots, z_n) \equiv \bar{u}$.
  - Indifference curve $U(y - b, \mathbf{z}) \equiv \bar{u}$ has MRS $b_{z_i}(\mathbf{z}, \bar{u}) = U_{z_i}/U_x$.
  - FOC: Bid function is tangent to the price function $b_{z_i} = p_{z_i}$
- Price function $p(\mathbf{z})$ is the upper envelope of the bid functions.

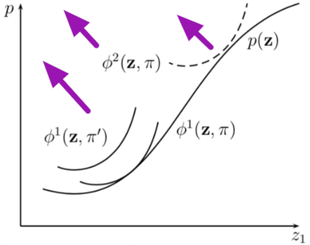


The Firm's Spatial Problem

- Rosen studies *short run equilibrium, fixing each firm's good* $\mathbf{z}$
- $C(Q, \mathbf{z})$ = cost of quantity Q of good $\mathbf{z} = (z_1, \ldots, z_n)$.
- In the *long run*, the firm chooses Q and $\mathbf{z}$ to maximize profits

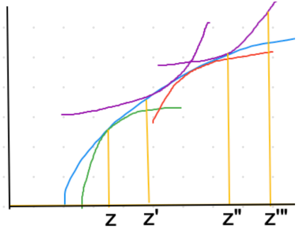
$$\max_{Q,\mathbf{z}} \Pi(p, Q, \mathbf{z}) = Qp(\mathbf{z}) - C(Q, \mathbf{z})$$

- Competition: *Firm takes the price function as given.*
    - FOC in Q: $p(\mathbf{z}) = C_Q(Q, \mathbf{z}) \Rightarrow$ supply function $Q^* = Q^*(p, \mathbf{z})$
    - FOC in z: $\Pi_{z_i}(p, Q^*, \mathbf{z}) = 0$ for all i yields $p_{z_i} = C_{z_i}/Q^*$.
- Offer function $\phi(\mathbf{z}, \bar{\pi})$ solves $\Pi(\phi(\mathbf{z}, \bar{\pi}), Q^*(p, \mathbf{z}), \mathbf{z}) \equiv \bar{\pi}$.
    - FOC: Offer function is tangent to the price function $b_{z_i} = p_{z_i}$
- Price function $p(\mathbf{z})$ is the lower envelope of the offer functions.



Market Equilibrium

- Market equilibrium is a *price function* $p(\mathbf{z})$, demand and supply densities $\delta(\mathbf{z}), \sigma(\mathbf{z})$ clearing the market: $\delta(\mathbf{z}) \equiv \sigma(\mathbf{z})$ for all $\mathbf{z}$.
- *Heterogeneity is essential*: The slope of the price function reflects the value of quality change of no particular consumer.
  - $p(z') - p(z)$ *overstates* value of quality change for consumers who buy z, and understates value of quality change for consumers who buy z'.
  - $p(z''') - p(z'')$ *understates* cost of quality improvement for producers who sell z'', and overstates cost of quality improvement for producers who sell z'''.



- Rosen solves a fun example but needs a differential equation (harder than our solving 1 equation in 1 unknown) $\Rightarrow$ beyond our math barrier
- Differential equations: computes bidding strategies in auctions (713B)

Two Location Hedonic Example

- To avoid differential equations, let's try two locales.
- Live next to the Capitol ($z = 1$), or far from it ($z = 0$)
- The competitive rent at $z = 0$ is fixed at $r > 0$
- There is an endogenous premium rent $R > r$ at $z = 1$
- Ms. θ has utility $U(x, z|\theta) = x + z/\theta$ over locale z & money x
  - Mass μ of residents has taste $1/\theta \in [0, \mu]$ for Capitol
  - We expect low θ residents live near Capitol, and high θ far
- Height h costs $C(h) = L + h^2$, given land cost premium $L > 0$.

- Hint: Put yourself in the model! Who will live where?

Offline: Hedonic Example Solution (Don't Peek!)

- Mass $\bar{\theta}$ of residents $\theta \in [0, \bar{\theta}]$ live at $z = 1$, for some $\bar{\theta} > 0$
- A spatial competitive equilibrium $(\bar{\theta}, h, L, R)$:
  (1) Buildings at $z = 1$ earn zero profits: $L + h^2 = C(h) = hR$
    - The Capitol location price premium
  (2) Price: Each building's height is optimal: $2h = C'(h) = R$
    - Production quantity: The Capitol location building height
  (3) *Resident type $\bar{\theta}$ is indifferent:* $R = r + 1/\bar{\theta}$
    - Optimal consumer allocation between locations
  (4) Apt. market clears at $z = 1$: $h = \bar{\theta} =$ resident mass in $[0, \bar{\theta}]$
    - Market clearing at Capitol location
- Solving the four equations in four unknowns:
  - Solution:
    $\sqrt{L} = r + \sqrt{r^2 + 8}$ & $\bar{\theta} = h = r + \sqrt{r^2 + 8}$ & $R = 2r + 2\sqrt{r^2 + 8}$
  - Derivation to check on your own:
    - From (1) and (2): $L = h^2 \Rightarrow h = \sqrt{L}$, $R = 2\sqrt{L}$
    - From (3): $1/\bar{\theta} = R - r = 2\sqrt{L} - r$
    - From (4): $\bar{\theta} = h = \sqrt{L}$

$\Rightarrow$ With higher land cost premium L, we have taller apartments, charging a higher rent premium R (Manhattan has tall buildings & big rents)

The Big Picture on the Pyramid of Giza, 1940

- The Great Pyramid of Giza has eight sides, not four.
- Each of the pyramid's four sides are evenly split from base to tip by concave indentations.

- Office half hours TuTh 2:30-3 after March break for prelim queries
- Come back when story-telling and modeling in your thesis!
- "Be proactive" (Habit 1 of The 7 Habits of Highly Effective People)