Introduction to Informational Herding

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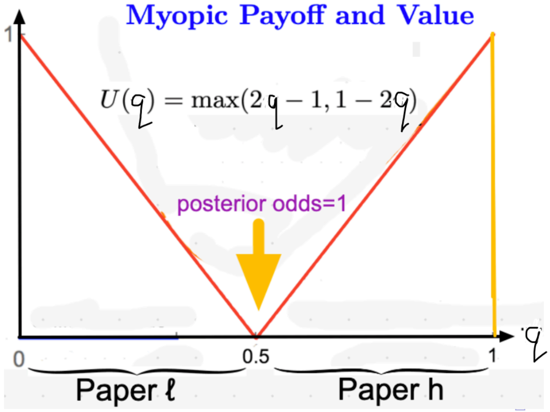
Communication by Actions: Informational Herding

- Common prior belief $\begin{cases} \pi \in (0,1) & \text{in state } \theta = H \\ 1-\pi & \text{in state } \theta = L \end{cases}$
- Unlike statistical learning, social learning can choke off endogenously, and can be mistaken (no law of large numbers).
- Infinitely many players acting in sequence in periods $t = 1, 2, 3, \ldots$, each first seeing all prior actions
  - *But players never see prior signals or payoffs*
- Individuals share identical payoffs over finitely many actions
- Everyone is endowed with a private signal σ with probability $f(\sigma|L)$ and $f(\sigma|H)$, conditionally independent of other signals
- With more signals than actions, $\nexists$ a separating equilibrium
- Even if not, the sequential equilibrium will entail pooling
- Eventually, it will hit perfect pooling, and choke off learning

A Very Simple Binary Action Decision Problem

- Example: Assume that economic theory research fashion is in the low-brow L or high-brow H state (*prior belief* π on H)
- Write a low-brow paper ℓ or high-brow paper h
- Research pays 1 if paper and state match, and -1 otherwise
  - $\Rightarrow$ *Expected payoff*: $\max(2q-1, 1-2q)$ for belief q



How Public Beliefs and Signals Lead to Actions

- *Public beliefs*: $\pi_1 = \pi$ & $\pi_t = P(H|a_1, \ldots, a_{t-1})$ for $t = 2, 3, \ldots$
- Use Bayes rule in likelihood ratio (LR) form:
  - Public likelihood ratio $\lambda_t = \pi_t/(1 - \pi_t)$ facing player t
  - Private signal likelihood ratios $\frac{f(\sigma|H)}{f(\sigma|L)}$
  - $\Rightarrow$ Player t's posterior q_t obeys: $\frac{q_t}{1-q_t} = \lambda_t \frac{f(\sigma_t|H)}{f(\sigma_t|L)}$
- Given posterior q_t, player t takes action $a_t = \begin{cases} \ell & \text{if } q_t \leq \frac{1}{2} \\ h & \text{if } q_t > \frac{1}{2} \end{cases}$
- Eg. Private signals $\sigma', \sigma'', \sigma'''$ chances $\begin{array}{l} f(\sigma_i|L) \propto \sqrt{2}, 1, \sqrt{2}/2 \\ f(\sigma_i|H) \propto \sqrt{2}/2, 1, \sqrt{2} \end{array}$
  - Likelihood ratio $\frac{f(\sigma|L)}{f(\sigma|H)} \in \{2, 1, \frac{1}{2}\}$.
- *How optimal actions reflect the public LR and private signals:*
  - $\lambda_t < 1/2 \Rightarrow q_t < \frac{1}{2}$ $\forall$ signals $\Rightarrow$ pool on action ℓ (a *cascade*)
  - $\lambda_t \in [1/2, 1) \Rightarrow [\sigma', \sigma'' \Rightarrow q_t < \frac{1}{2} \rightarrow \ell$ and $\sigma''' \Rightarrow q_t \geq \frac{1}{2} \rightarrow h]$
  - $\lambda_t \in [1, 2) \Rightarrow [\sigma' \Rightarrow q_t < \frac{1}{2} \rightarrow \ell$ and $\sigma'', \sigma''' \Rightarrow q_t \geq \frac{1}{2} \rightarrow h]$
  - $\lambda_t \geq 2 \Rightarrow q_t \geq \frac{1}{2}$ $\forall$ signals $\Rightarrow$ pool on action h (a *cascade*)
- Why? Social learning stops in a cascade, since the public belief overwhelms all private signals

Informational Herding: Learning from the Last Action

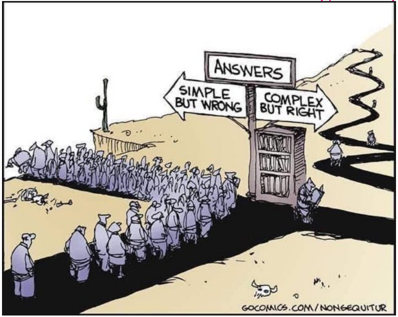
- Assume no cascade: $1/2 \leq \lambda_t < 2$ at period t
- How to compute the next public LR, accounting for pooling:
  - $\lambda_t \in [1, 2) \rightarrow \begin{cases} \text{Given signal } \sigma', \text{ we take action } \ell \\ \text{Given signal } \sigma'' \text{ or } \sigma''', \text{ we take action } h \end{cases}$
  - If player $t+1$ sees action $a_t = h$, then $\lambda_{t+1} = \sqrt{2}\lambda_t$
    - Proof: Player $t+1$ infers $\{\sigma'' \cup \sigma'''\}$
    - The likelihood ratio of $\{\sigma'' \cup \sigma'''\}$ is $\frac{1+\sqrt{2}}{1+\sqrt{2}/2} = \sqrt{2}$
  - Comment: After seeing actions $a_t = a_{t+1} = h$, we get $\lambda_{t+2} = \sqrt{2}\lambda_{t+1} = (\sqrt{2})^2\lambda_t \geq 2 \Rightarrow$ cascade on action h
    - So even if true state is $\theta = L$, with positive probability, the first two signals are σ'', σ'', and so we take action h forever
  - If player $t+1$ sees $a_t = \ell$, he infers $\sigma_t = \sigma' \Rightarrow \lambda_{t+1} = \frac{1}{2}\lambda_t$
- The case $\lambda_t \in [\frac{1}{2}, 1)$ is left as an exercise.

Famous Advanced Theory Topic: Informational Cascades

- Bikhchandani, Hirshleifer, & Welch (1992),"A Theory of Fads, Fashion, Custom, & Cultural Change as Informational Cascades"
- Banerjee (1992), "A Simple Model of Herd Behavior"
  - Assume an infinite sequence of people act in sequence.
  - Theorem: *Eventually, a cascade starts on some action, and and thus action herd starts. With positive probability, that herd is not on the highest payoff action*
  - Comment: With more signals than actions, $\nexists$ separating eq'm.
  - But this holds even for binary signals
- Pieter Bruegel the Elder, 1568: "The Blind Leading the Blind"
  - Misleading! Rather it is the seeing *rationally acting as if blind*

Informational Cascades: Too Good to Be Generally True

- Multinomial signals is a simple but misleading example.
- Assume a general signal (e.g. infinitely many outcomes?)
- My imprint on this problem was Smith and Sorensen (2000), "Pathological Outcomes of Observational Learning"
  - **Theorem.** If the *signal likelihood ratio* $f^H(\sigma)/f^L(\sigma)$
    - $\uparrow \infty$ for some $\sigma \Rightarrow$ people learn with certainty if the state is H
    - $\downarrow 0$ for some $\sigma \Rightarrow$ people learn with certainty if the state is L
  - Plot twist: *a cascade need never happen!* But we always end in a herd: *eventually, everyone chooses the same action.*

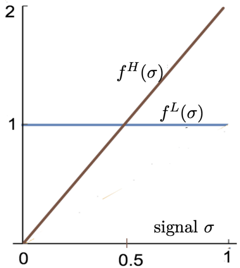


Irony: Cascades themselves were simple but wrong!

Bonus Informational Herding Solved Homework Exercise

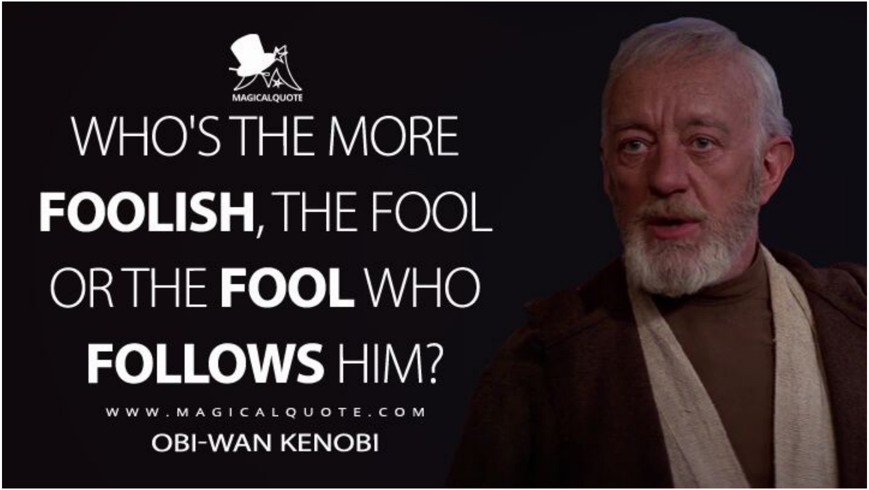
- Assume same states and payoffs as before
- Private signals are conditionally independent *densities* $f^H(\sigma) = 2\sigma$ & $f^L(\sigma) = 1$ on $[0, 1]$

Signal Densities



- Question: For which public beliefs is there a cascade?
- Question: If only the first player is informed, what is the expected payoff of all players?
- Can you plot it as a function of the public belief π?

Bonus Informational Herding Solution (Don't Peak)



Bonus Informational Herding Solved Homework Exercise

- Since $f^H(\sigma)/f^L(\sigma) = 2\sigma$, higher signals σ favor state H.
- Write paper h iff $q \geq 1/2$, or posterior odds ≥ 1
  - $\Leftrightarrow$ posterior odds $2\sigma[\pi/(1-\pi)] \geq 1$.
  - $\Leftrightarrow$ private signal $\sigma \geq \bar{\sigma}(\pi) \equiv (1-\pi)/(2\pi)$
  - For $\pi \leq 1/3$, we always have $\sigma < \bar{\sigma}(\pi) \Rightarrow$ we take action ℓ
  - So the cascade set for action ℓ is $\pi \in [0, 1/3]$
  - The cascade set for action h is just $\pi = 1$ (i.e. trivial)
- Intuitively, once the public belief in H drops below $1/3$, no private signal can push the posterior over $1/2$,
  - $\Rightarrow$ one is guaranteed to take action ℓ.
  - $\Rightarrow$ public beliefs remains unchanged, as no new information arrives
- If state is H, social learning won't reveal it, and we forever take the wrong action.

Bonus Informational Herding Solved Homework Exercise

- With just one informed player, the expected payoff is, if $\pi < 1/3$:
$$V(\pi) = \max(2\pi - 1, 1 - 2\pi)$$
while if $\pi \geq 1/3$, we have (with $F^H(\sigma) = \sigma^2$ and $F^L(\sigma) = \sigma$):
$$\begin{aligned} V(\pi) &= \pi[1 - 2F^H(\bar{\sigma}(\pi))] + (1-\pi)[2F^L(\bar{\sigma}(\pi)) - 1] \\ &= \pi[1 - 2((1-\pi)/(2\pi))^2] + (1-\pi)[2(1-\pi)/(2\pi) - 1] \\ &= \tfrac{1}{2}(5\pi - 4 + 1/\pi). \end{aligned}$$

  i.e. "With chance π, the state is H, and I get expected payoff $[1 - F^H(\bar{\sigma}(\pi))] - F^H(\bar{\sigma}(\pi))$. With chance $1-\pi$, the state is L..."